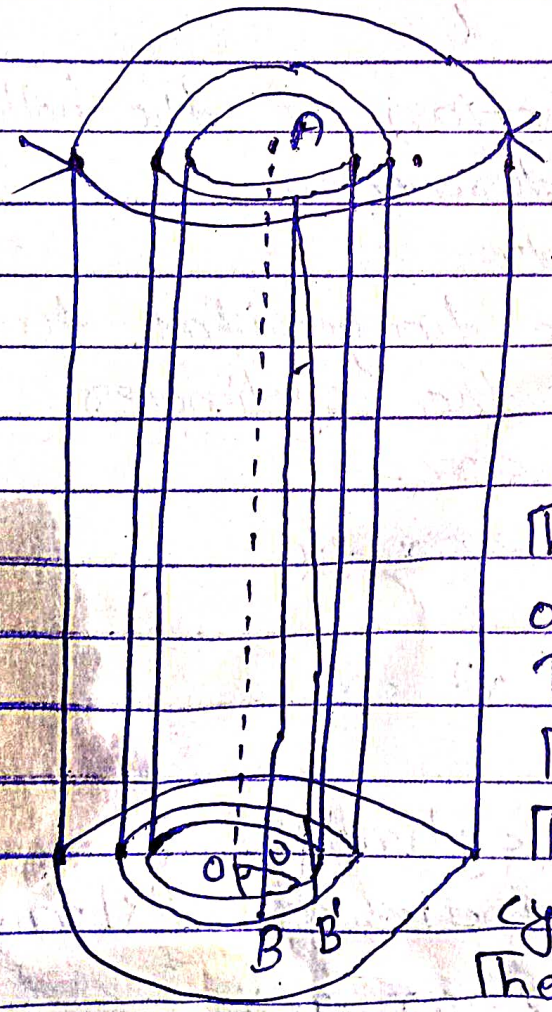
Torque on a Solid Cylinder :-

Let us consider a solid cylinder of length l and radius r which is clamped At the upper end rigidly. Let it be Twisted through an angle θ by means of a torque. let the cylinder be made of radius x and thickness dx Let AB be a line on its surface Initialy perpendicular to the Fixed end. after the cylinder is twisted let B moved to B' A will



Not change because it is A point on the clamped End then $BOB' = \theta$ Will be called the angle of twist hence the angle of twist is the angle Through Which a radius of the free end is Turned $BAB' = \phi$ is The shearing strain The surface of the hollow cylinder where as θ is The Same for all the Hollow cylinder, ϕ changes from cylinder to cylinder it is being

Maximum at the outermost surface And Zero at the axis OO

From the figure $BB' = l\phi = x\theta$

$$\text{or, } \phi = \frac{x\theta}{l}$$

Let f be the shearing stress on The surface of this cylinder

$$\text{Than } \eta \text{ (Rigidity)} = \frac{f}{\phi} = \frac{fl}{x\theta}$$

$$\text{or, } f = \frac{\eta x \theta}{l}$$

Area of cross section of the hollow cylinder in consideration

$$= 2\pi x \cdot dx$$

∴ Shearing force tangential to The surface of the cylinder

$$= \text{Stress} \times \text{area}$$

$$= \frac{\eta x \theta}{l} \times 2\pi x \, dx$$

$$= \frac{2\pi\eta\theta}{l} x^2 \, dx$$

Torque on the cylinder =

force x perpendicular distance

$$= \frac{2\pi\eta\theta}{l} x^2 dx \times x$$

$$= \frac{2\pi\eta\theta}{l} x^3 dx$$

∴ Torque on the solid cylinder = Summation of torques on hollow cylinders

$$\frac{2\pi n\theta}{l}\int_{x=0}^{x=r} x^3\,dx = \frac{2\pi n\theta}{l}\left[\frac{x^4}{4}\right]_0^r$$

$$\frac{2\pi n\theta}{l}\cdot\frac{r^4}{4} = \frac{n\pi r^4}{2l}\theta$$

When θ = 1 radian

$$\text{Torque } (\tau) = \frac{n\pi r^4}{2l}$$

This is called torsional rigidity Thus torsional rigidity or modulus of torsion of a cylinder is a The torque required to twist Through 1 radian it is usually denoted by C or I.

Thus I (torque on a cylinder) = $C\theta$ where C is the torsional rigidity

Thus the couple required per unit Twist in case of cylinder of length and radius r will be

$$(C) = \frac{n\pi r^4}{2l}$$